LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - **STATISTICS**

FOURTH SEMESTER – **APRIL 2013**

ST 4811 - ADVANCED OPERATIONS RESEARCH

Date : 30/04/2013 Dept. No. Max. : 100 Marks
Time : 1:00 - 4:00

SECTION A

Answer ALL questions. Each carries two marks. **(10 X 2 = 20)**

1. Define a General Linear Programming Problem.

2. Define a general primal dual pair.

3. What is goal programming? State its applications.

4. State the principal of optimality in dynamic programming.

5. What is the need for integer programming?

6. What is a Non Linear Programming Problem?

7. Define a quadratic programming problem.

8. What is inventory control?

9. If one arrival occurs every 10 minutes, determine the average arrival rate per hour, and the average inter-arrival time in hours.

10. What is a deterministic queueing system?

SECTION B

Answer any FIVE questions. Each carries eight marks. **(5 X 8 = 40)**

11. Use two-phase simplex method to maximize $Z = 5\,X_1 + 3\,X_2$, subject to the constraints,

$2X_1 + X_2 \le 1$; $X_1 + 4\,X_2 \ge 6$; and X_1 , $X_2 \ge 0$.

12. Describe the Gomory's constraint method, and derive Gomory's constraint for solving a Pure Integer Programming Problem.

13. State the necessary conditions for solving the following Quadratic programming Problem.

Max $Z = 6\,X_1 + 3\,X_2 - 4\,X_1\,X_2 - 2\,X_1^2 - 3\,X_2^2$ subject to the constraints,

$X_1 + X_2 \le 1$; $2\,X_1 + 3\,X_2 \le 4$; and X_1 , $X_2 \ge 0$, and show that Z is strictly concave.

14. Test for extreme values of $f(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2$, subject to the constraints,

$x_1 + x_2 + 3x_3 = 2$ and $5x_1 + 2x_2 + x_3 = 5$.

15. Using Dynamic Programming Problem, maximize z = $\{y_1.y_2.....y_n\}$ subject to the constraints, $y_1 + y_2 + ... + y_n = c$, and $y_j \geq 0$.

16. A corporation is entertaining proposals from its 3 plants for possible expansion of its facilities. The corporation's budget is £ 5 millions for allocation to all 3 plants. Each plant is requested to submit its proposals giving total cost C and total revenue R for each proposal. The following table summarizes the cost and revenue in millions of pounds. The zero cost proposals are introduced to allow for the probability of not allocating funds to individual plants. The goal of the corporation is to maximize the total revenue resulting from the allocation of £ 5 millions to the three plants.

	Plant 1		Plant 2		Plant 3	
Proposal	C_1	R_1	C_2	R_2	C_3	R_3
1	0	0	0	0	0	0
2	1	5	2	8	1	3
3	2	6	3	9	-	-
4	-	-	4	12	-	-

Use Dynamic Programming Problem to obtain the optimal policy for the above problem.

17. Explain the classical static Economic Order Quantity model and derive the expressions for Total Cost per Unit, order quantity, ordering cycle and effective lead time.

18. Explain the important characteristics of a queuing system.

SECTION C

Answer any TWO questions. Each carries twenty marks. **(2 X 20 = 40)**

19. Maz $Z = 200\ X_1 + 400\ X_2 + 300\ X_3$, subject to the constraints,

$30\ X_1 + 40\ X_2 + 20\ X_3 \leq 600$

$20\ X_1 + 10\ X_2 + 20\ X_3 \leq 400$

$10\ X_1 + 30\ X_2 + 20\ X_3 \leq 800$,

X_1 , X_2 , X_3 are non-negative integers.

Introducing slack variables and using simplex method, a non-integer optimum solution is given below;

C	X_B	X_0	X_1	X_2	X_3	X_4	X_5	X_6
400	X_2	20/3	1/3	1	0	1/3	-1/3	0
300	X_3	50/3	5/6	0	1	-1/6	2/3	0
0	X_6	80/3	-5/3	0	0	-2/3	-1/3	1
	Z-C	23000/3	550/3	0	0	250/3	200/3	0

Find an OBFIS (Optimum Basic Feasible Integer Solution) to the above problem.

20. Solve the following Non Linear Programming Problem: Max $Z = 7\ X_1^2 + 6\ X_1 + 5\ X_2^2$ subject to the constraints, $X_1 + 2\ X_2 \leq 10$; $X_1 - 3\ X_2 \leq 9$; and X_1 , $X_2 \geq 0$,

21. Solve the following Quadratic programming Problem, by Wolfe's algorithm.

Max $Z = 4\ X_1 + 6\ X_2 - 2\ X_1\ X_2 - 2\ X_1^2 - 2\ X_2^2$ subject to the constraints,

$X_1 + 2\ X_2 \leq 2$; X_1 , $X_2 \geq 0$.

22. (i) Neon lights in an industrial park are replaced at the rate of 100 units per day. The physical plant orders the neon lights periodically. It costs Rs. 100 to initiate a purchase order. A neon light kept in storage is estimated to cost about Re. 0.02 per day. The lead time between placing and receiving an order is 12 days. Determine the optimum inventory policy for ordering the neon lights.

(ii) For a (M/M/1) : (∞/FIFO) queuing model in the steady-state case, derive the steady state difference equations and obtain expressions for the mean and variance of queue length in terms of the parameters λ and μ. (10 + 10)

*****